Appendix D
Laplace Transform Table

TO ACCOMPANY

AUTOMATIC CONTROL SYSTEMS
EIGHTH EDITION

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APPENDIX D

Laplace Transform Table

Laplace Transform $F(s)$	Time Function $f(t)$
1	Unit-impulse function $\delta(t)$
$\frac{1}{s}$	Unit-step function $u_s(t)$
$\frac{1}{s^2}$	Unit-ramp function t
$\frac{n!}{s^{n+1}}$	$t^n (n = \text{positive integer})$
$\frac{1}{s+\alpha}$	$e^{-\alpha t}$
$\frac{1}{(s+\alpha)^2}$	$te^{-\alpha t}$
$\frac{n!}{(s+\alpha)^{n+1}}$	$t^n e^{-\alpha t} (n = \text{positive integer})$
$\frac{1}{(s+\alpha)(s+\beta)}$	$\frac{1}{\beta-\alpha}(e^{-\alpha t} - e^{-\beta t})(\alpha \neq \beta)$
$\frac{s}{(s+\alpha)(s+\beta)}$	$\frac{1}{\beta-\alpha}(\beta e^{-\beta t} - \alpha e^{-\alpha t})(\alpha \neq \beta)$
$\frac{1}{s(s+\alpha)}$	$\frac{1}{\alpha}(1 - e^{-\alpha t})$
$\frac{1}{s(s+\alpha)^2}$	$\frac{1}{\alpha^2}(1 - e^{-\alpha t} - \alpha t e^{-\alpha t})$
$\frac{1}{s^2(s+\alpha)}$	$\frac{1}{\alpha^2}(\alpha t - 1 + e^{-\alpha t})$
$\frac{1}{s^2(s+\alpha)^2}$	$\frac{1}{\alpha^2}\left[t - \frac{2}{\alpha} + \left(t + \frac{2}{\alpha}\right)e^{-\alpha t}\right]$
$\frac{s}{(s+\alpha)^2}$	$(1 - \alpha t)e^{-\alpha t}$

(continued)

Laplace Transform $F(s)$	Time Function $f(t)$
$\dfrac{\omega_n}{s^2 + \omega_n^2}$	$\sin \omega_n t$
$\dfrac{s}{s^2 + \omega_n^2}$	$\cos \omega_n t$
$\dfrac{\omega_n^2}{s(s^2 + \omega_n^2)}$	$1 - \cos \omega_n t$
$\dfrac{\omega_n^2(s + \alpha)}{s^2 + \omega_n^2}$	$\omega_n \sqrt{\alpha^2 + \omega_n^2}\, \sin(\omega_n t + \theta)$ where $\theta = \tan^{-1}(\omega_n/\alpha)$
$\dfrac{\omega_n}{(s + \alpha)(s^2 + \omega_n^2)}$	$\dfrac{\omega_n}{\alpha^2 + \omega_n^2} e^{-\alpha t} + \dfrac{1}{\sqrt{\alpha^2 + \omega_n^2}} \sin(\omega_n t - \theta)$ where $\theta = \tan^{-1}(\omega_n/\alpha)$
$\dfrac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$	$\dfrac{\omega_n}{\sqrt{1 - \zeta^2}} e^{-\zeta\omega_n t} \sin \omega_n \sqrt{1 - \zeta^2}\, t \qquad (\zeta < 1)$
$\dfrac{\omega_n^2}{s(s^2 + 2\zeta\omega_n s + \omega_n^2)}$	$1 - \dfrac{1}{\sqrt{1 - \zeta^2}} e^{-\zeta\omega_n t} \sin(\omega_n \sqrt{1 - \zeta^2}\, t + \theta)$ where $\theta = \cos^{-1}\zeta \qquad (\zeta < 1)$
$\dfrac{s\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$	$\dfrac{-\omega_n^2}{\sqrt{1 - \zeta^2}} e^{-\zeta\omega_n t} \sin(\omega_n \sqrt{1 - \zeta^2}\, t - \theta)$ where $\theta = \cos^{-1}\zeta \qquad (\zeta < 1)$
$\dfrac{\omega_n^2(s + \alpha)}{s^2 + 2\zeta\omega_n s + \omega_n^2}$	$\omega_n \sqrt{\dfrac{\alpha^2 - 2\alpha\zeta\omega_n + \omega_n^2}{1 - \zeta^2}} e^{-\zeta\omega_n t} \sin(\omega_n \sqrt{1 - \zeta^2}\, t + \theta)$ where $\theta = \tan^{-1} \dfrac{\omega_n \sqrt{1 - \zeta^2}}{\alpha - \zeta\omega_n} \qquad (\zeta < 1)$
$\dfrac{\omega_n^2}{s^2(s^2 + 2\zeta\omega_n s + \omega_n^2)}$	$t - \dfrac{2\zeta}{\omega_n} + \dfrac{1}{\omega_n \sqrt{1 - \zeta^2}} e^{-\zeta\omega_n t} \sin(\omega_n \sqrt{1 - \zeta^2}\, t + \theta)$ where $\theta = \cos^{-1}(2\zeta^2 - 1) \qquad (\zeta < 1)$