

Appendix D

Laplace Transform Table

TO ACCOMPANY
AUTOMATIC CONTROL SYSTEMS
EIGHTH EDITION

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ISBN 0-471-13476-7

Laplace Transform Table

| Laplace Transform $F(s)$ | Time Function $f(t)$ |
|-------------------------------------|--|
| 1 | Unit-impulse function $\delta(t)$ |
| $\frac{1}{s}$ | Unit-step function $u_s(t)$ |
| $\frac{1}{s^2}$ | Unit-ramp function t |
| $\frac{n!}{s^{n+1}}$ | t^n ($n = \text{positive integer}$) |
| $\frac{1}{s + \alpha}$ | $e^{-\alpha t}$ |
| $\frac{1}{(s + \alpha)^2}$ | $te^{-\alpha t}$ |
| $\frac{n!}{(s + \alpha)^{n+1}}$ | $t^n e^{-\alpha t}$ ($n = \text{positive integer}$) |
| $\frac{1}{(s + \alpha)(s + \beta)}$ | $\frac{1}{\beta - \alpha}(e^{-\alpha t} - e^{-\beta t})$ ($\alpha \neq \beta$) |
| $\frac{s}{(s + \alpha)(s + \beta)}$ | $\frac{1}{\beta - \alpha}(\beta e^{-\beta t} - \alpha e^{-\alpha t})$ ($\alpha \neq \beta$) |
| $\frac{1}{s(s + \alpha)}$ | $\frac{1}{\alpha}(1 - e^{-\alpha t})$ |
| $\frac{1}{s(s + \alpha)^2}$ | $\frac{1}{\alpha^2}(1 - e^{-\alpha t} - \alpha t e^{-\alpha t})$ |
| $\frac{1}{s^2(s + \alpha)}$ | $\frac{1}{\alpha^2}(\alpha t - 1 + e^{-\alpha t})$ |
| $\frac{1}{s^2(s + \alpha)^2}$ | $\frac{1}{\alpha^2}\left[t - \frac{2}{\alpha} + \left(t + \frac{2}{\alpha}\right)e^{-\alpha t}\right]$ |
| $\frac{s}{(s + \alpha)^2}$ | $(1 - \alpha t)e^{-\alpha t}$ |

(continued)

| Laplace Transform $F(s)$ | Time Function $f(t)$ |
|--|---|
| $\frac{\omega_n}{s^2 + \omega_n^2}$ | $\sin \omega_n t$ |
| $\frac{s}{s^2 + \omega_n^2}$ | $\cos \omega_n t$ |
| $\frac{\omega_n^2}{s(s^2 + \omega_n^2)}$ | $1 - \cos \omega_n t$ |
| $\frac{\omega_n^2(s + \alpha)}{s^2 + \omega_n^2}$ | $\omega_n \sqrt{\alpha^2 + \omega_n^2} \sin(\omega_n t + \theta)$ where $\theta = \tan^{-1}(\omega_n/\alpha)$ |
| $\frac{\omega_n}{(s + \alpha)(s^2 + \omega_n^2)}$ | $\frac{\omega_n}{\alpha^2 + \omega_n^2} e^{-\alpha t} + \frac{1}{\sqrt{\alpha^2 + \omega_n^2}} \sin(\omega_n t - \theta)$ where $\theta = \tan^{-1}(\omega_n/\alpha)$ |
| $\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$ | $\frac{\omega_n}{\sqrt{1 - \zeta^2}} e^{-\zeta\omega_n t} \sin \omega_n \sqrt{1 - \zeta^2} t \quad (\zeta < 1)$ |
| $\frac{\omega_n^2}{s(s^2 + 2\zeta\omega_n s + \omega_n^2)}$ | $1 - \frac{1}{\sqrt{1 - \zeta^2}} e^{-\zeta\omega_n t} \sin(\omega_n \sqrt{1 - \zeta^2} t + \theta)$ where $\theta = \cos^{-1} \zeta \quad (\zeta < 1)$ |
| $\frac{s\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$ | $\frac{-\omega_n^2}{\sqrt{1 - \zeta^2}} e^{-\zeta\omega_n t} \sin(\omega_n \sqrt{1 - \zeta^2} t - \theta)$ where $\theta = \cos^{-1} \zeta \quad (\zeta < 1)$ |
| $\frac{\omega_n^2(s + \alpha)}{s^2 + 2\zeta\omega_n s + \omega_n^2}$ | $\omega_n \sqrt{\frac{\alpha^2 - 2\alpha\zeta\omega_n + \omega_n^2}{1 - \zeta^2}} e^{-\zeta\omega_n t} \sin(\omega_n \sqrt{1 - \zeta^2} t + \theta)$ where $\theta = \tan^{-1} \frac{\omega_n \sqrt{1 - \zeta^2}}{\alpha - \zeta\omega_n} \quad (\zeta < 1)$ |
| $\frac{\omega_n^2}{s^2(s^2 + 2\zeta\omega_n s + \omega_n^2)}$ | $t - \frac{2\zeta}{\omega_n} + \frac{1}{\omega_n \sqrt{1 - \zeta^2}} e^{-\zeta\omega_n t} \sin(\omega_n \sqrt{1 - \zeta^2} t + \theta)$ where $\theta = \cos^{-1}(2\zeta^2 - 1) \quad (\zeta < 1)$ |